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Responsibility sensitive egalitarianism and optimal linear income taxation

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Abstract

We compute optimal linear taxes on labor income with quasilinear preferences between income and labor. Agents differ in their productivity and in their taste for leisure. A responsibility sensitive egalitarian wants to compensate for the former differences but not for the latter. This intuition is captured by a social planner that wants to equalize opportunities for subjective utility along the lines of the criteria proposed by Roemer and Van de gaer, and by a social planner evaluating social states based on an advantage function representing reference preferences. Our theoretical results are illustrated with empirical data for Belgium.

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1. Introduction

The problem of optimal income taxation when people have different preferences for leisure raises difficult normative questions. A higher income may be due either to differences in innate productivity and skill levels, or to differences in the degree of effort. Progressive taxes can therefore imply redistribution from those with a low preference for leisure to those with a high preference for leisure. The ethical evaluation of this result may

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depend on the exact interpretation given to the preference parameter. One may have ethical objections against a redistribution from the energetic to the lazy. Things are different, however, if the higher preference for leisure is linked to lower physical or mental abilities to work. These different intuitions are linked to the notion of responsibility. In general, many people feel that some redistribution is legitimate because people should be compensated for factors which are beyond their control. Because innate skills and productive endowments are a prominent example of the latter, this leads us directly into the traditional literature on optimal income taxation. At the same time, however, they also feel that people should be held responsible for factors which are under their control. This consideration is largely absent from the traditional optimal tax literature. In a setting with responsibility, the tradeoff between equity and efficiency becomes a trilemma, involving efficiency, compensation and responsibility. In this paper, we formulate a model of optimal linear income taxation to analyze some aspects of this trilemma.

It is not obvious how to reconcile the different concerns. Problems arise if we want to hold individuals *fully* accountable for differences in outcomes that result from differences in (pure) preferences (e.g., for work effort), while we want to compensate them *fully* for skill (or ability) differences. [Fleurbaey \(1995a,b\)](#) and [Bossert \(1995\)](#) have proven that these two principles (“equal transfers for equal skills” and “equal income for equal preferences”, respectively) are not compatible in general. [Bossert and Fleurbaey \(1996\)](#) have shown similar incompatibilities within the specific problem of optimal (first best) income taxation. Thus, several suggestions have been made that involve some tradeoff between the two principles. A general idea in this literature is the choice of so-called reference preferences or reference skills. The social objective keeps one of the two principles intact, and ensures that the other intuition holds true for the reference preferences or the reference level of skills (see, e.g., [Bossert and Fleurbaey, 1996](#)). [Fleurbaey and Maniquet \(2004a\)](#) develop social orderings that incorporate compensation for inequalities in skills and responsibility for preferences, and derive optimal taxes within this framework—see, e.g., [Fleurbaey and Maniquet \(2002\)](#). The social orderings they introduce use information on the indifference curves of the individuals but remain completely ordinal.

These axiomatic approaches are somewhat different in spirit from the traditional optimal taxation literature. Recently, several authors have analyzed in a more traditional way the design of optimal income taxes when agents have different preferences. [Sandmo \(1993\)](#) shows that the case for redistributive taxation from the rich towards the poor is weakened in a model with a utilitarian planner if the rich, because of their lower preference for leisure, are more efficient at the margin in generating utility. [Boadway et al. \(2001\)](#) analyze the problem of optimal nonlinear income taxation when the government can observe the income level of its citizens, but not their skills or preferences. To simplify the analysis, they assume preferences to be quasilinear and more specifically linear in leisure.¹ They consider utilitarian social orderings where different weights are attached to different preferences and where reference preferences are used. An alternative approach is followed by [Roemer et al. \(2003\)](#). They implement the concept of equality of opportunity ([Roemer, 1998](#)). “Equal income for equal preferences” means that, ideally, an individual’s income

¹In particular, they specify preferences as $u(c)-\alpha L$ with $\alpha > 0$, where c is consumption and L is labor supply.

should not depend on his level of skills. Because this has to hold for all preferences in society, it will generally not be possible to achieve this. Therefore, Roemer suggests that we maximize a weighted average of the minimal utilities across individuals having the same tastes. In their optimal taxation application, [Roemer et al. \(2003\)](#) assume preferences to be quasilinear and more specifically linear in consumption.² Moreover, their social objective function is not defined in terms of utilities but in terms of income. This can be interpreted as a special case of reference preferences.

In this paper, we want to compare explicitly different egalitarian approaches. Each of them embodies a variant of the maximin criterion. To simplify the analysis and to obtain explicit solutions for the optimal tax rates, we concentrate on the case of linear income taxation. Moreover, we follow [Atkinson \(1995\)](#) and [Roemer et al. \(2003\)](#) in assuming that preferences are quasilinear (linear in income) and yield an isoelastic labor supply curve. The general structure of our model is described in Section 2.

From Section 3 onwards, we compute optimal tax rates based on social welfare functions that differ in two dimensions; they are subjective or objective and look at outcomes or opportunities. As pointed out by [Sen \(1991\)](#), social welfare functions can be based on subjective utilities or on a more objective concept of well-being. To reflect the latter, we propose to use for social evaluation purposes an advantage function which is meant to represent the living standard (the “good”) of the individual. Basically, the choice of this advantage function boils down to the choice of a reference preference ordering between consumption and leisure which differs from individual preferences. Opportunity egalitarianism is captured by two variants of the idea of “equality of opportunity”. In addition to Roemer’s criterion, we present and analyze a related proposal that maximizes the average utility of the skill group for which average utility is lowest ([Van de gaer, 1993](#)). Section 3 analyzes subjective outcome egalitarianism. This amounts to classical welfarist egalitarianism. Next we analyze nonwelfarist taxes as follows. Section 4 analyzes the subjective opportunity case, Section 5 looks into the objective outcome case and Section 6 discusses the objective opportunity optimal taxes. Section 7 contains an empirical illustration for Belgium. Section 8 concludes.

2. The model

To represent the problem of responsibility vs. compensation as simply as possible, we assume that individuals differ in only two dimensions. The first dimension is their skill level w , assumed to be beyond their control because it is linked to their genetic endowment. The second variable is a preference parameter e , meant to capture a pure preference for leisure. In a certain sense, it represents the degree of diligence. We deliberately interpret the variables in such a way that our ethical intuitions imply that compensation is desirable for differences in w , while at the same time individuals can be held responsible for differences in e .³ We suppose that both variables have finite support;

²Preferences are specified as $c\text{-}xv(L)$ (see also [Diamond, 1998](#) for an optimal taxation exercise with these preferences). [Roemer et al. \(2003\)](#) assume in addition that labor supply is isoelastic.

³Differences in physical and mental abilities are assumed to be subsumed in the skill level w .

the preference parameter and the skill level are measured such that $0 < e_L \leq e \leq 1$ and $0 < w_L \leq w \leq 1$, respectively. This assumption of finite support will allow us to identify the worst-off individual later on. Moreover, we assume that e and w are distributed independently with density functions $f_w(w) : [w_L, 1] \rightarrow \mathbb{R}_+$ and $f_e(e) : [e_L, 1] \rightarrow \mathbb{R}_+$. This independence assumption simplifies the technical aspects of the problem. Moreover, the interpretation of responsibility for e becomes rather tricky when preferences are correlated with w , interpreted as genetic endowment.

We work in a second-best context, in which the social planner does not observe the individual w and e but knows the density functions. While she can observe labor income, she cannot determine whether it is due to differences in genetic skill endowments or to differences in preferences. To focus on the problem of responsibility sensitivity, we limit ourselves to the case of linear income taxation with a constant marginal tax rate t and a lump sum grant B . This lump sum grant B could be interpreted as the level of basic income. Moreover, we only consider the case of an egalitarian government. As usual, we suppose that efficiency is desirable and we interpret egalitarianism as maximin. The crux of the paper is the comparison of the results for different interpretations of this egalitarian starting point.

2.1. Individual behavior

We assume a quasilinear form for preferences between income Y and labor supply L :

$$u(Y, L) = Y - \frac{1}{e} \frac{\varepsilon}{1 + \varepsilon} (L_0)^{-\frac{1}{\varepsilon}} L^{\frac{1+\varepsilon}{\varepsilon}} \quad (1)$$

with $L_0 > 0$ and $\varepsilon \geq 0$. L_0 is the maximal amount of work someone can perform, and ε is the constant elasticity of labor supply, a measure for the efficiency cost of the tax and assumed to be identical for all individuals.⁴ The marginal rate of substitution between income and labor is given by $(1/e)(L/L_0)^{(1/\varepsilon)}$ and is therefore dependent on the idiosyncratic taste parameter e . It will be equal to zero when $L=0$ and equal to $1/e$ when $L=L_0$. The quasilinear specification (Eq. (1)) implies that the marginal rates of substitution for two individuals with different tastes for leisure will always be a constant multiple of each other. Therefore, indifference curves of two individuals will satisfy the single crossing property.⁵

In the context of linear income taxation, after tax income consists of the lump sum grant received from the government, B , and labor income after taxes, $(1-t)wL$:

$$Y = B + (1 - t)wL. \quad (2)$$

⁴Because ε is identical for all individuals, it is irrelevant whether we consider it as a responsibility or a compensation variable. People will be held responsible for differences in labor supply only when these follow from differences in e .

⁵An alternative specification would introduce taste variety through ε . In that case, the ratio of the marginal rates of substitution between two individuals depends on (L/L_0) . Indifference curves still satisfy the single crossing property. We do not follow this specification because the identification of the worst-off would become much more difficult and may depend on the level of taxes.

Substituting the budget constraint (Eq. (2)) in the utility function (Eq. (1)) yields

$$U(L; B, t) = B + (1 - t)wL - \frac{1}{e} \frac{\varepsilon}{1 + \varepsilon} (L_0)^{-\frac{1}{\varepsilon}} L^{\frac{1+\varepsilon}{\varepsilon}} \tag{3}$$

Maximization of Eq. (3) yields the isoelastic labor supply:

$$L = (e(1 - t)w)^\varepsilon L_0 \tag{4}$$

Note that, if $t < 1$, labor supply will be positive, while, if t is positive, labor supply will be smaller than L_0 .⁶ From Eq. (4), it is clear that the supply of labor is an increasing function of e and w . Those with the smallest disutility of labor (the largest e) and the highest level of skills will have the biggest labor supply.

Preference satisfaction can be measured by the indirect utility function,

$$V(e, w, B, t) = B + L_0(1 - t)^{1+\varepsilon} w^{1+\varepsilon} e^\varepsilon \frac{1}{1 + \varepsilon} \tag{5}$$

This expression immediately shows that utility is increasing in both w and e . People with a higher preference for leisure work less, but the resulting increase in their leisure time does not compensate for the smaller income they get. This is not an innocuous cardinalization. As a matter of fact, interpersonal comparisons of utility are extremely tricky in this situation of differences in preferences. If people are aware of Eq. (5) and control the preference parameter e , why would they not opt for a larger value if this leads to a larger utility level? We return to this question in Section 5, in which we introduce the advantage function.

2.2. The government revenue constraint

Using Eq. (4), the revenue constraint faced by the government is given by⁷

$$B(t) = L_0 t (1 - t)^\varepsilon \int_{e_L}^1 e^\varepsilon f_e(e) de \int_{w_L}^1 w^{1+\varepsilon} f_w(w) dw \tag{6}$$

This expression can be written in terms of the moments of the distributions of e and w . We define the α th moment of a variable x with support $[\underline{x}, \bar{x}]$ as $\mu_\alpha(x) = \int_{\underline{x}}^{\bar{x}} x^\alpha f(x) dx$. Using this definition, we rewrite Eq. (6) as:

$$B(t) = L_0 t (1 - t)^\varepsilon \mu_\varepsilon(e) \mu_{1+\varepsilon}(w) \tag{7}$$

⁶It will turn out that the socially optimal tax rates are between 0 and 1.

⁷Adding a requirement of financing public goods would not change anything if public goods enter the utility function in an additively separable way.

For later reference it is useful to derive the revenue-maximizing tax rate t_{BI} . Differentiation of Eq. (7) immediately shows that

$$\frac{t_{BI}}{1 - t_{BI}} = \frac{1}{\varepsilon}$$

We use the subscript BI to indicate that this is also the tax rate which would maximize basic income (see [Atkinson, 1995](#) for a similar interpretation).

3. Subjective outcome egalitarianism: the welfarist maximin benchmark

An egalitarian government that is only concerned with the preference satisfaction of its citizens will maximize the utility of the individual that is least well off.⁸ Formally, this government tries to maximize

$$\min_{e,w} V(e, w, B, t) \tag{8}$$

subject to Eq. (7). From Eq. (5), it is clear that the least well off in terms of indirect utility is always the individual with characteristics (e_L, w_L) . A subjective outcome egalitarian government will thus maximize $V(e_L, w_L, B, t)$.⁹ This immediately suggests an awkward question: is it ethically acceptable to tax everybody (even the hard working but low-skilled persons) to raise the utility of the laziest persons in society? The problem gets more concrete when we derive the resulting optimal level of taxes. This follows easily after we have introduced Eq. (7) into Eq. (5).

Proposition 1. *The subjective outcome egalitarian optimal tax rate $t_{E(V)}$ is defined by*

$$\frac{t_{E(V)}}{1 - t_{E(V)}} = \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon} e_L^\varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \right]$$

Note that this tax rate is smaller than t_{BI} . This is easily interpreted because, in our model, the worst-off individual still has a positive labor supply and therefore the tax imposes a welfare cost on him. The optimal tax rate has to balance this welfare cost against the effect of taxes on the level of $B(t)$. More importantly for our purposes, Proposition 1 also shows that $t_{E(V)}$ depends on e_L . If we compare different distributions with the same

⁸While it has become common practice to label this objective function Rawlsian, it is clear that [Rawls \(1971\)](#) never advocated it. An individual's well-being should be measured in terms of his primary goods, not in terms of his level of preference satisfaction. His ideas are therefore closer to the "advantage" approach developed in Section 5.

⁹Note that this result again follows from our specific cardinalization of the utility function.

$\mu_e(e)$ but with different values for e_L , the optimal marginal tax rate will increase if e_L goes down. If the laziest individual in society gets lazier, the tax rate on all the others increases. This may seem an undesirable result. We will now explore the consequences of some alternative social objective functions, which try to capture the idea that people are responsible for their degree of diligence.

4. Subjective opportunity egalitarianism¹⁰

Let us first keep the subjective starting point that, for the social evaluation, individuals' subjective utility (Eq. (5)) matters. However, an "opportunity" egalitarian will not simply maximize Eq. (8), but will take the position that individuals should only be compensated for utility differences following from differences in their productivity, i.e., their innate skill level w , while at the same time held responsible for differences in their preferences or their diligence e . One possible approach to this problem has been introduced as "equality of opportunity" by Roemer (1998). Roemer proposes to partition the set of agents in subsets with the same value of the nonresponsibility factor w . These subsets are called "types". For persons with the same type, differences in e (and hence L) will lead to different outcomes. According to Roemer, this is not a problem from an opportunity viewpoint. However, differences in outcomes for individuals with the same value for e should be avoided because these differences can only follow from differences in innate skills w . He therefore suggests the following social objective function:

$$\eta_{I(V)} = \int_{e_L}^1 f_e(e) \min_w V(e, w, B, t) de \quad (9)$$

In this function, the egalitarian (maximin) idea is applied at each e -level separately, and afterwards a simple sum is taken over all possible e -levels.¹¹

While Roemer's proposal is well known, an obvious alternative was proposed by Van de gaer (1993). He suggests to specify the social objective function as

$$\eta_{S(V)} = \min_w \int_{e_L}^1 f_e(e) V(e, w, B, t) de \quad (10)$$

This proposal is easily understood once we follow the suggestion made by Bossert et al. (1999) to interpret the distribution of possible outcomes for a given type w as the

¹⁰Roemer (2002) emphasizes that an opportunity approach is not "welfarist", as welfarism is defined as an approach which requires only knowledge of the utility possibilities set to compute the optimal policy.

¹¹This formulation deviates slightly from Roemer's original formulation. He incorporates in his framework an identification axiom which says that people have exercised the same degree of responsibility if they are at the same percentile of the distribution of effort within their type. For our model, however, with the same distribution of 'effort' for all types, the two formulations are equivalent.

opportunity set S_w of that type. Focusing on utility as the relevant outcome, the opportunity set of an individual with productivity w can be written as

$$S_w = \{(O, e) \in \mathbb{R} \times [e_L, 1] \mid O = V(e, w, B, t)\} \tag{11}$$

Fig. 1 depicts two such opportunity sets for individuals of different types. The social objective function (Eq. (10)) evaluates the area under these opportunity sets for the different types, where the different e -levels are weighted by their density. In a certain sense, this can be interpreted as computing the “average utility” obtained by individuals of a given type. It then considers the worst-off type as the type with the smallest area under his opportunity set and maximizes this area (compare Eqs. (8) and (10)). Roemer’s objective function (Eq. (9)) looks at the intersection of the areas under opportunity sets and maximizes that intersection. This explains our choice of the subscripts “S” (for smallest) and “I” (for intersection), respectively. Both criteria are equivalent if the opportunity sets do not intersect. This is the case in our subjective model, because the indirect utility function (Eq. (5)) is monotonically increasing both in w and in e . Fig. 1 is therefore a little misleading; in the subjective approach of this section, the opportunity set of the type $w = w_L$ will lie below the opportunity set of all other skill levels.¹² In this paper, we do not go deeply into the normative comparison of these two approaches.¹³ Note that they both reduce to utilitarianism in the case where there is only one skill level w . They both reduce to subjective outcome egalitarianism if there is no variation in the preference parameter e . It is thought that in general $\eta_{I(V)}$ has a more egalitarian flavor than $\eta_{S(V)}$ (see, e.g., [Bossert et al., 1999](#)).

It is straightforward to calculate the optimal tax rates in this setting. The maximization of Eq. (9) and of Eq. (10) using Eqs. (5) and (7) results in

Proposition 2. *The subjective opportunity egalitarian optimal tax rate is defined by*

$$\frac{t_{I(V)}}{1 - t_{I(V)}} = \frac{t_{S(V)}}{1 - t_{S(V)}} = \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon}}{\mu_{1+\varepsilon}(w)} \right]$$

Because $e_L^e \leq \mu_e(e)$, we can derive immediately the following corollary

Corollary 1. *Subjective outcome egalitarian taxes are always at least as high as subjective opportunity egalitarian taxes.*

This corollary is in line with the general intuition that introducing opportunity considerations in general will lead to lower optimal tax rates. Note also that the distribution of the preference parameter e does no longer play any role in the expression

¹²This will no longer be true in Section 6, where we introduce opportunity in an objective egalitarian framework.

¹³But see, e.g., the discussion in [Fleurbaey and Maniquet \(2004b\)](#) and [Ooghe et al. \(2003\)](#).

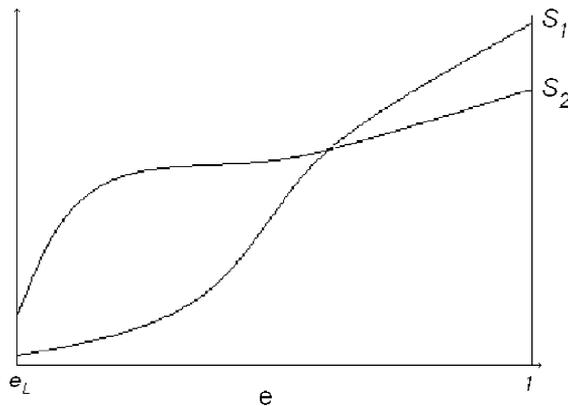


Fig. 1. “Opportunity” sets.

for the optimal tax rate $t_{l(v)}$ [$=t_{s(v)}$].¹⁴ If all individuals have the same skill level, the optimal tax rates become zero.

5. Objective outcome egalitarianism

Introducing responsibility considerations in an explicit way is one approach to the dilemmas raised by classical welfarist egalitarianism. An alternative is to keep egalitarianism but to drop subjectivism, i.e., the starting point that individual utilities matter. The limitations of subjectivism are by now well understood (see, e.g., [Dworkin, 1981a](#)). Important points of criticism have been the treatment of expensive tastes and the overly subjective and idiosyncratic nature of individual preferences. In a certain sense, the problem of different preferences for leisure as sketched in the previous sections is a good illustration of this basic weakness. The literature contains some ambitious alternative proposals: [Rawls \(1971\)](#) proposes the notion of “primary goods”, [Dworkin \(1981b\)](#) advocates the idea of “equality of resources”, [Sen \(1980\)](#) the idea of “basic capabilities”, [Cohen \(1990\)](#) that of “midfare”. We will be less ambitious and focus on the most crucial aspect in the context of the problem of optimal income taxation: the preference for leisure. Our “objective” well-being concept remains close to the individual utility function (Eq. (5)), but incorporates the idea of society or of the social planner about what is a “reasonable” preference for leisure. The resulting “objective” well-being concept will be denoted the “advantage” of the individuals. In Section 5.1, we will first present the specific form and the characteristics of our advantage function and then return to its interpretation. Section 5.2 gives the results for the optimal taxes. Section 5.3 discusses the potential conflict between objective egalitarianism and Pareto efficiency.

¹⁴We will discuss in Section 6 the differences between this model and the one proposed in [Roemer et al. \(2003\)](#).

5.1. *The advantage function*

Following [Vandenbroucke \(2001\)](#), we assume that the government evaluates individual advantage by the function:

$$a(Y, L) = Y - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} (L_0)^{-\frac{1}{\varepsilon} L^{\frac{1+\varepsilon}{\varepsilon}}} \tag{12}$$

The parameter g represents what we called the “reasonable” preference for leisure, i.e., the government’s policy stance w.r.t. individual well-being or socially desirable effort levels. As g increases, the burden of market work, as perceived by the social planner, decreases. If g becomes infinite, only income matters. This is the special case analyzed in [Roemer et al. \(2003\)](#). Although this special case may not seem very “reasonable”, income considerations (both in terms of maximizing aggregate income and minimizing income inequality) have traditionally played a dominant role in real world policy debates. Recently, however, there has been a growing trend in many countries to consider explicitly the tradeoff between, on the one hand, material welfare and economic growth, and on the other hand, nonmaterial quality of life. The parameter g captures this tradeoff, a decrease in g corresponding to a larger weight given to nonmaterial quality of life, relative to material welfare (income).

We can use the individual budget constraint (Eq. (2)) and the labor supply function (Eq. (4)) to write advantage as a function of the characteristics of individuals and of the policy parameters:

$$A(e, w, B, t) = B + L_0(1 - t)^{1+\varepsilon} w^{1+\varepsilon} e^\varepsilon \frac{1}{1 + \varepsilon} \left[1 + \varepsilon \left(1 - \frac{e}{g} \right) \right] \tag{13}$$

From a comparison of Eq. (13) with Eq. (5), we see immediately that

$$V(e, w, B, t) \geq (\leq) A(e, w, B, t) \Leftrightarrow e \geq (\leq) g$$

This stands to reason since those with $e \geq (\leq) g$ work too much (not enough) according to the advantage function. However, this interpretation depends on the specific cardinalization of the indirect utility function. This cardinalization is not necessary when we work with the advantage function. To understand better the properties of the latter, it is useful to differentiate Eq. (13) partially w.r.t. w and e . This yields

Lemma 1. *Properties of the advantage function:*

$$(1) \frac{\partial A(e, w, B, t)}{\partial w} \geq 0 \Leftrightarrow e \leq g \frac{1 + \varepsilon}{\varepsilon}$$

$$(2) \frac{\partial A(e, w, B, t)}{\partial e} \geq 0 \Leftrightarrow e \leq g$$

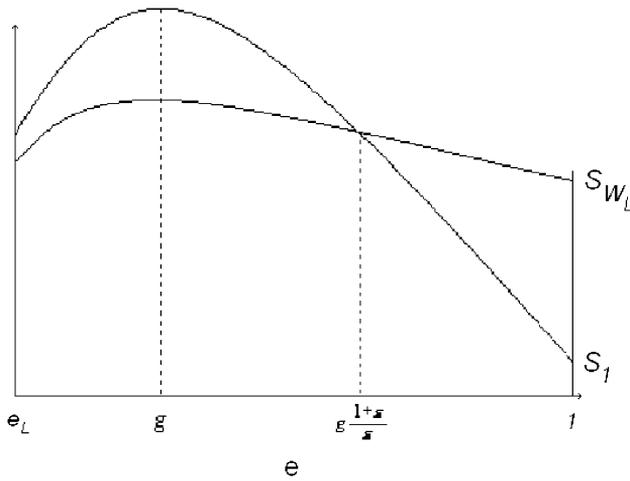


Fig. 2. Properties of the advantage function.

The findings of the lemma are illustrated in Fig. 2. For a given type w , the advantage function reaches its maximum where $e = g$, i.e., for those individuals whose preference for leisure coincides with the “social” preference for leisure. Society ascribes a lower advantage to individuals with $e \neq g$, the more e differs from g . To compare the advantage functions of different types (i.e., different values of w), note first that they all cross for $e = g \frac{1+\epsilon}{\epsilon}$. For that specific value of e , the advantage function reduces to B and is therefore completely independent of labor supply and labor income. To the left of the intersection point, individuals with a larger w reach a higher advantage level; to the right of the intersection point, the opposite is true. This follows from the fact that more productive individuals work more (see Eq. (4)). If this is combined with a large value of e (to the right of the intersection point), they are working “too much” according to the social preferences, so that their higher labor income no longer compensates for the increased effort (in the eyes of the social planner).¹⁵

The previous lemma leads directly to the following corollaries:¹⁶

Corollary 2. Consider all individuals with a given preference parameter \bar{e} . The lowest advantage is ascribed to individuals with skills $w = w_L$ or $w = 1$. More specifically, we have three cases:

Case 1: $\bar{e} < g \frac{1+\epsilon}{\epsilon} \Leftrightarrow w_L = \operatorname{argmin} A(\bar{e}, w, B, t) (\forall B, t)$

Case 2: $\bar{e} = g \frac{1+\epsilon}{\epsilon} \Leftrightarrow A(\bar{e}, w_L, B, t) = A(\bar{e}, 1, B, t) = A(\bar{e}, w, B, t) \forall w \in [w_L, 1], (\forall B, t)$

Case 3: $\bar{e} > g \frac{1+\epsilon}{\epsilon} \Leftrightarrow 1 = \operatorname{argmin} A(\bar{e}, w, B, t) (\forall B, t)$

¹⁵In the special case treated by Roemer et al. (2003), in which $g = \infty$, the advantage functions for all types are increasing in e over the observed range $e_L \leq e \leq 1$ and at any given level of e the individual with the lowest productivity level also has the lowest advantage. This stands to reason because $g = \infty$ implies that society only considers income and does not care about effort at all.

¹⁶The proofs of Corollary 3 and of all the following propositions are given in Appendix A.

Corollary 3. Consider all individuals of a given type \bar{w} . The lowest advantage is ascribed to individuals with preferences $e = e_L$ or $e = 1$. More specifically, we have three cases:

Case 1: $g < \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Leftrightarrow 1 = \operatorname{argmin} A(e, \bar{w}, B, t) (\forall B, t)$

Case 2: $g = \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Leftrightarrow A(e_L, \bar{w}, B, t) = A(1, \bar{w}, B, t) \leq A(e, \bar{w}, B, t) \forall e \in [e_L, 1], (\forall B, t)$

Case 3: $g = \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Leftrightarrow e_L = \operatorname{argmin} A(e, \bar{w}, B, t) (\forall B, t)$

An objective egalitarian social planner will maximize the advantage of the individual that is worst-off in terms of advantage. Formally, she maximizes

$$\varphi_{E(A)} = \min_{e,w} A(e, w, B, t) \tag{14}$$

The identification of the worst-off requires some care. Indeed, as the previous results show, for an objective egalitarian it is no longer necessarily true that the least well off in terms of advantage is always the person with the highest disutility from labor and the lowest level of skills. More specifically, the identification of the least well off in terms of advantage depends on the social disutility of labor g (the policy stance with respect to preferences for leisure) and the elasticity of labor supply ε . We identify the least well off in the following proposition, which is illustrated in the upper panel of Fig. 3.

Proposition 3. The objective egalitarian objective:

Case 1: $g > \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Rightarrow \varphi_{E(A)} = A(e_L, w_L, B, t)$

Case 2: $g = \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Rightarrow \varphi_{E(A)} = A(e_L, w_L, B, t) = A(1, w_L, B, t)$

Case 3: $\frac{\varepsilon}{1 + \varepsilon} < g < \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Rightarrow \varphi_{E(A)} = A(1, w_L, B, t)$

Case 4: $g = \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \varphi_{E(A)} = B$

Case 5: $g < \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \varphi_{E(A)} = A(1, 1, B, t)$

Depending on the value of g , different individuals will be considered to be in the worst-off position. It is only when g is large enough that the smallest advantage level will be ascribed to the lazy low-wage individuals. If the government attaches a high disadvantage to work $g < \varepsilon/(1 + \varepsilon) \equiv \alpha$, individuals with a low disutility from work and high wages are considered to be worst-off. In a certain sense, they work too much. For intermediate values of g , the lowest advantage is ascribed to the hard working individuals with low productivity. Note that the intelligent but lazy individuals are never worst-off with the advantage function we have specified. Note also that in the specific case 4, the objective function coincides with the maximization of basic income, as could be expected from Lemma 1.

$g \rightarrow$		αe_L		$\alpha \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)}$	α		$\alpha \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}$	
$\varphi_{E(A)}$	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)		(1, w_L)	(e_L, w_L)
$\varphi_{S(A)}$	\bar{A}_1	\bar{A}_1	\bar{A}_1		\bar{A}_L	\bar{A}_L	\bar{A}_L	\bar{A}_L
$\varphi_{I(A)}$	\bar{A}_1		MIX	MIX	MIX		\bar{A}_L	\bar{A}_L

NOTE: $\alpha = \frac{\varepsilon}{1 + \varepsilon}$

Fig. 3. Determination of the worst-off for different values of g .

Let us now return to the different interpretations which can be given to this “objective egalitarian” approach in terms of advantage. In a first interpretation, our advantage function could be seen as an index of primary goods. Traditionally, Rawlsian justice measures advantage in terms of primary goods, including income and wealth. The neglect of leisure is a well-known problem in Rawlsian justice, first highlighted by [Musgrave \(1974\)](#). Our model could be seen as an admittedly primitive way of including leisure in the set of Rawlsian primary goods. In a second interpretation, g can be seen as the choice of a reference value for e in Eq. (1). We then come close to the tradition in the social choice literature of picking a set of reference preferences. In the context of our problem, an immediate interpretation would be that this parameter g simply reflects the conception of the social planner. In a somewhat broader perspective, one could interpret these reference preferences or this value of g as a kind of social norm, close to [Scanlon’s \(1982\)](#) idea of moral justification. In this approach, people can only claim an income if they can defend towards others in society that they have performed a “reasonable” amount of effort. As a matter of fact, we can leave it open where the concrete value of the parameter g comes from and do sensitivity analysis with respect to different values.

The basic idea of introducing the notion of a “reasonable” preference for leisure is to introduce a distinction between “subjective preferences” on the one hand and “objective well-being” on the other hand. This approach could be seen as paternalist, in the sense that it goes against the principle of absolute consumer sovereignty. On the other hand, the government does not impose any behavior on the economic agents. In this model, they all take their decisions based on their own preferences (Eq. (5)), leading to the labor supply function (Eq. (4)). The social planner is paternalist only in the sense that she uses the advantage function to define the optimal values of the tax instruments; these instruments do determine the economic environment in which agents have to take their own decisions, but the agents remain completely free in their own choices. As we will see, maximization of Eq. (14) does not necessarily lead to a Pareto-efficient tax rate if we interpret Pareto efficiency in terms of indirect utility. We will return to that problem in Section 5.3.

5.2. Optimal objective outcome egalitarian taxation

To derive these optimal taxes, we substitute the government revenue constraint (Eq. (7)) in the objective function of the government (Eq. (14)) and maximize the resulting expression.

Proposition 4. *The objective outcome egalitarian optimal tax rates are as follows.*

$$\text{Case 1: } g > \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Rightarrow \frac{t_{E(A)}}{1 - t_{E(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{e_L^\varepsilon w_L^{1+\varepsilon}}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left(1 - \frac{e_L}{g} \right) \right] \right]$$

$$\begin{aligned} \text{Case 2: } g = \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} &\Rightarrow \frac{t_{E(A)}}{1 - t_{E(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{(1 + \varepsilon) w_L^{1+\varepsilon}}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 - \frac{e_L^\varepsilon - 1}{e_L^{1+\varepsilon} - 1} \right] \right] \\ &= \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon}}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left(1 - \frac{1}{g} \right) \right] \right] \end{aligned}$$

$$\text{Case 3: } \frac{\varepsilon}{1 + \varepsilon} < g < \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} \Rightarrow \frac{t_{E(A)}}{1 - t_{E(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon}}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left(1 - \frac{1}{g} \right) \right] \right]$$

$$\text{Case 4: } g = \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \frac{t_{E(A)}}{1 - t_{E(A)}} = \frac{1}{\varepsilon}$$

$$\text{Case 5: } g < \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \frac{t_{E(A)}}{1 - t_{E(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{1}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left(1 - \frac{1}{g} \right) \right] \right]$$

The resulting pattern of tax rates as a function of g is summarized in Fig. 4. It stands to reason that the expression for the optimal tax rate depends on g . As g increases, we

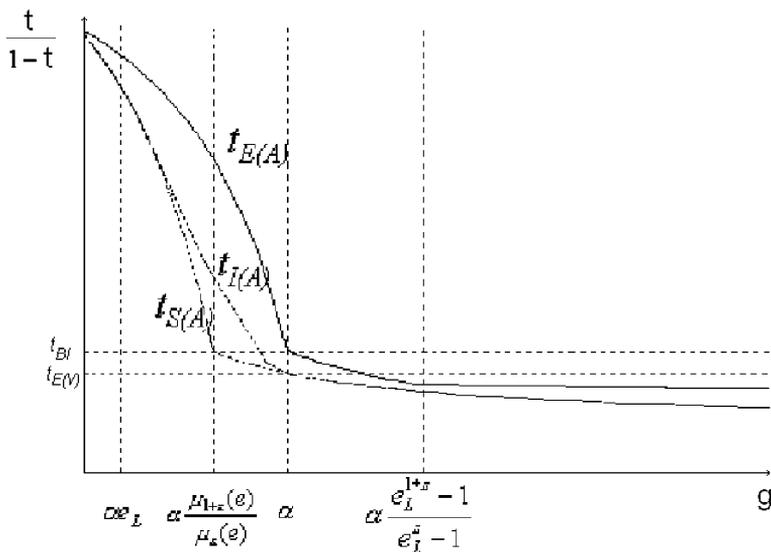


Fig. 4. Optimal tax rates for different values of g .

move gradually from case 5, case 4, case 3, case 2, down to case 1, implying a shift in the definition of who is the least advantaged person (see Proposition 3) and a decrease in the optimal tax rate. Moreover, within each of the cases described in the proposition, taxes decrease as the government attaches a smaller disadvantage to labor (i.e., as g increases). The slope of $[t_{E(A)}/(1-t_{E(A)})]$ as a function of g becomes less negative when we move from case 5 down to case 3 all over case 4 because $w_L < 1$, and when we go from case 3 down to case 1 all over case 2 because $e_L < 1$. Therefore, the negatively sloped tax function has kinks at the points where $g = \varepsilon/(1 + \varepsilon)$, and $g = \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}$. At these kink points, the slope of $[t_{E(A)}/(1-t_{E(A)})]$ as a function of g becomes less negative. The negative effect of increasing g on the optimal tax rate is easily understood; increasing g implies that the social planner attaches a lower disadvantage to labor which she therefore wants to discourage less.

Additional insights into the structure of these optimal taxes come from comparing the results for case 1 of Proposition 4 with their counterpart in Proposition 1. In this case 1, the worst-off individual in terms of advantage has the same characteristics as the worst-off in the subjective: (e_L, w_L) . Objective egalitarian taxes are then equal to subjective egalitarian taxes, except for the term $\varepsilon[1 - (e_L/g)]$. This factor disappears when $e_L = g$, in which case the advantage function coincides with the indirect utility function of the laziest person in society. In all other cases, the term $\varepsilon[1 - (e_L/g)]$ can be interpreted as an “objective” (advantage) correction factor of the subjective egalitarian tax. If $g > e_L$, the advantage cost of labor is smaller than the disutility experienced by the individual that is worst-off in terms of advantage. The correction factor $\varepsilon[1 - (e_L/g)]$ is then positive and brings the optimal tax rate down, thereby inducing the worst-off to work more. The other cases described in Proposition 4 have a correction factor that can be interpreted similarly. There are thus two reasons why objective egalitarian and subjective egalitarian taxes differ: the correction factor and the identification of the worst-off.

It is worthwhile comparing the consequences of the two approaches we have followed until now. Introducing opportunity egalitarianism in a subjective framework unambiguously reduces the optimal tax rate as compared to the simple outcome egalitarian approach (see Corollary 1). This is not true when we take an objective egalitarian point of view. Indeed, as the previous discussion has shown, the relative ranking of subjective and objective outcome egalitarian taxes depends on the magnitude of the parameter g , i.e., on the attitude of the social planner towards preferences for leisure. For low values of g , the optimal tax rate in the objective egalitarian setting is larger than the subjective egalitarian optimal tax rate. This immediately raises the question of the Pareto efficiency of the solution.

5.3. Objective egalitarianism and Pareto efficiency

Many economists, while sympathetic towards the idea of “objective egalitarianism”, are at the same time reluctant to give up the traditional notion of Pareto efficiency, in which the domination criterion is defined in terms of subjective preferences (see, e.g., [Gaspard, 1998](#)). Let us therefore analyze the issue of second-best Pareto efficiency in our model.

Starting from Eqs. (5) and (7), we derive that an individual with characteristics (w_i, e_i) will prefer a tax rate

$$\frac{t_i}{1 - t_i} = \frac{1}{\varepsilon} \left(1 - \frac{w_i^{1+\varepsilon} e_i^\varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \right) \quad (15)$$

As a matter of fact, this is an obvious generalization of Proposition 1. Eq. (15) shows that there is a direct relationship between $w_i^{1+\varepsilon} e_i^\varepsilon$ and the desired tax rate; individuals with a larger value of $w_i^{1+\varepsilon} e_i^\varepsilon$ (and hence, as follows immediately from Eq. (4), a larger labor income) will prefer a lower tax rate t_i . Moreover, it is easy to see that $\partial V_i / \partial t \leq 0$ for $t \geq t_i$; that is, individual preferences are single-peaked over the t -dimension. It follows that a tax rate cannot be Pareto efficient if it is larger than the tax rate preferred by the individual (w_L, e_L) , i.e., the subjective egalitarian tax rate $t_{E(V)}$, or smaller than the tax rate preferred by the individual $(1, 1)$, i.e.,

$$\frac{t_{\text{MIN}}}{1 - t_{\text{MIN}}} = \frac{1}{\varepsilon} \left(1 - \frac{1}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \right)$$

Let us now return to objective egalitarianism and look at the results of Proposition 4 as summarized in Fig. 4. It is very well possible that for large values of g we would get $t_{E(A)} < t_{\text{MIN}}$.¹⁷ This is a fortiori true for the case $g \rightarrow \infty$ (analyzed in [Roemer et al., 2003](#)). Whether this will happen, depends on the specific values taken by ε , e_L and w_L . More interesting in our setting is the possibility that $t_{E(A)} > t_{E(V)}$. This Pareto-inefficient result can be obtained for a large range of g -values. In case 1, $t_{E(A)} > t_{E(V)}$ if $g < e_L$. One may doubt that the planner would pick a “reasonable” value for g outside the range of observed e -values.¹⁸ However, there are also complications in the other cases. The optimal objective egalitarian tax rate is certainly Pareto inefficient in cases 4 and 5 [with $g \leq \varepsilon/(1 + \varepsilon)$, where it is larger than or equal to t_{B1} . It is also Pareto inefficient in case 3 for $g < \varepsilon/(1 + \varepsilon - e_L^\varepsilon)$.

The question arises how to interpret this finding. In the first place, one could raise the problem of political feasibility. It seems unrealistic to expect a social planner to introduce a Pareto-inefficient tax rate going against the preferences of all individuals in society. However, once one takes a political point of view, the whole optimal taxation exercise gets less relevant. After all, because our model implies single-peaked preferences over the t -dimension, we can derive immediately that the median voter (the individual with median gross wage income) will be decisive and will opt for Eq. (15) for the median value of $w_i^{1+\varepsilon} e_i^\varepsilon$. Our model deliberately neglects issues of political feasibility to concentrate on the ethical tradeoffs of the social planner. Turning then to these ethical issues, it seems that one

¹⁷It follows from a comparison of t_{MIN} with case 1 in Proposition 4 that this will occur if $e_L^\varepsilon w_L^{1+\varepsilon} \times (1 + \varepsilon(1 - \frac{e_L}{g})) > 1$.

¹⁸Moreover, a necessary condition to have $e_L > g$ in case 1 would be $e_L > \frac{\varepsilon}{1+\varepsilon}$. For reasonable values of ε , this implies a restricted support for e .

can take two positions. The first would be accepting the ethical primacy of Pareto efficiency in terms of preferences, and hence the ethical unacceptability of Pareto-inefficient taxes. One could then introduce Pareto efficiency as a side constraint in the maximization problem of the social planner and restrict the range of acceptable tax rates to $[t_{\text{MIN}}, t_{\text{E}(V)}]$. Or one could at a more basic level question the choice of a value of $g < \varepsilon / (1 + \varepsilon - e_L^\varepsilon)$ in the advantage function. Both these approaches are rather ambiguous, however. From the point of view of principles, it seems that one either accepts subjectivism (and then the introduction of an advantage function is not desirable), or one rejects subjectivism—and then it is no longer obvious that Pareto efficiency in terms of subjective preferences is desirable.¹⁹

6. Objective opportunity egalitarianism

Until now we have followed two approaches to tackle the problem of different preferences for leisure. First, we introduced the notion of opportunity egalitarianism in a subjective setting. Second, we turned to a so-called objective egalitarian framework by introducing the concept of a “reasonable” (reference) preference for leisure through the advantage function. We can also combine the two approaches and introduce opportunity egalitarianism in the objective framework. This makes sense if we interpret the “advantage” function as a description of the objective individual well-being and argue that individuals themselves can be held responsible for their own preferences towards leisure.

6.1. Objective opportunity egalitarian objectives

The criteria proposed by Roemer, φ_I , and Van de gaer, φ_S , are now redefined as

$$\varphi_{I(A)} = \int_{e_L}^1 f_e(e) \min_w A(e, w, B, t) de \tag{16}$$

$$\varphi_{S(A)} = \min_w \int_{e_L}^1 f_e(e) A(e, w, B, t) de \tag{17}$$

These criteria can again be interpreted in terms of opportunity sets. In the present context, they reflect the opportunity for advantage. The opportunity set for advantage of an individual with productivity w can be determined as

$$S_w = \{(O, e) \in \mathbb{R} \times [e_L, 1] \mid O = A(e, w, B, t)\} \tag{18}$$

As before, $\varphi_{I(A)}$ looks at the intersection of the areas under the opportunity sets, while $\varphi_{S(A)}$ looks at the smallest area under the sets. For later reference, it is useful to write down

¹⁹Note, e.g., that Gaspart (1998) explicitly argues that individual preferences must be laundered for Pareto efficiency to be acceptable.

the expressions for the areas of the opportunity sets of the most skilled and least skilled individuals, respectively, using Eqs. (13) and (17):

$$\bar{A}_1(t; g) = B + L_0(1 - t)^{1+\varepsilon} \left[\mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{1+\varepsilon}(e) \right] \tag{19}$$

$$\bar{A}_L(t; g) = B + L_0(1 - t)^{1+\varepsilon} w_L^{1+\varepsilon} \left[\mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{1+\varepsilon}(e) \right] \tag{20}$$

The criteria $\varphi_{I(A)}$ and $\varphi_{S(A)}$ are equivalent if the opportunity sets do not intersect. While such intersections did not occur in the subjective setting, they are possible now. Not surprisingly, the occurrence of intersections depends on the value of g . More specifically, we can derive

Proposition 5. *The objective opportunity egalitarian objectives:*

Case 1: $g \geq \frac{\varepsilon}{1 + \varepsilon} e_L \Rightarrow \varphi_{I(A)} = \varphi_{S(A)} = \bar{A}_1(t; g)$

Case 2: $\frac{\varepsilon}{1 + \varepsilon} e_L < g < \frac{\varepsilon}{1 + \varepsilon} e \Rightarrow \varphi_{I(A)} = B + L_0(1 - t)^{1+\varepsilon} C[w_L, e_L, g, \varepsilon; f_e(e)]$ with

$$C(\cdot) = w_L^{1+\varepsilon} \int_{e_L}^{g \frac{1+\varepsilon}{\varepsilon}} e^\varepsilon \left[1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right] f_e(e) de + \int_{g \frac{1+\varepsilon}{\varepsilon}}^1 e^\varepsilon \left[1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right] f_e(e) de$$

(a) $g < \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \varphi_{S(A)} = \bar{A}_1(t; g)$

(b) $g = \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \varphi_{S(A)} = B$

(c) $g > \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \varphi_{S(A)} = \bar{A}_L(t; g)$

Case 3: $g \geq \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \varphi_{I(A)} = \varphi_{S(A)} = \bar{A}_L(t; g)$

These results are summarized in the lower panel of Fig. 3. With subjective opportunity egalitarianism, those with the lowest skills always have the smallest opportunity set (see Section 4). This is no longer true when we adopt objective egalitarianism and introduce an advantage function. This is easily understood when remembering the structure of the advantage function in Fig. 2 and Lemma 1. There we have noticed that advantage functions cross for $e = g[(1 + \varepsilon)/\varepsilon]$. Because the support of e is given by $[e_L, 1]$, it immediately follows that we will not have a relevant crossing in cases 1 and 3 of Proposition 5. If the disadvantage from work is high (g is small), the highly skilled will be working too much in the eye of the ethical observer. Their opportunity set will lie below all other opportunity sets and $\varphi_{I(A)}$ and $\varphi_{S(A)}$ coincide; this is case 1. As the disadvantage of labor decreases, g increases, and the opportunity sets offered to workers with different skill levels cross (case 2). $\varphi_{I(A)}$ and $\varphi_{S(A)}$ become different; $\varphi_{I(A)}$ looks at $S_1 \cap S_{wL}$, while $\varphi_{S(A)}$ looks at the smallest opportunity set. The highest skilled worker will have the smallest opportunity set as long as $g < [\varepsilon/(1 + \varepsilon)] \{[\mu_{1+\varepsilon}(e)]/[\mu_\varepsilon(e)]\}$, and the lowest skilled thereafter. If $g = [\varepsilon/(1 + \varepsilon)] \{[\mu_{1+\varepsilon}(e)]/[\mu_\varepsilon(e)]\}$, the two coincide and the social planner wants to

maximize basic income. As the disadvantage attached to work keeps decreasing, we enter case 3; the opportunity set of the lower skilled, S_{wL} , will lie below all other opportunity sets and $\varphi_{I(A)}$ and $\varphi_{S(A)}$ will coincide once more.

6.2. Objective opportunity egalitarian optimal taxes

Using the same methodology as before, we can now compute the optimal taxes. The resulting pattern is shown in Fig. 4.

Proposition 6. *The objective opportunity egalitarian optimal taxes are defined as follows.*

$$\text{Case 1: } g \leq \frac{\varepsilon}{1 + \varepsilon} e_L \Rightarrow \frac{t_{I(A)}}{1 - t_{I(A)}} = \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{1}{\mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left[1 - \frac{1}{g} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \right] \right] \right]$$

$$\text{Case 2: } \frac{\varepsilon}{1 + \varepsilon} e_L < g < \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \frac{t_{I(A)}}{1 - t_{I(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{(1 + \varepsilon)}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} C(\cdot) \right]$$

$$(a) \ g < \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{1}{\mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left[1 - \frac{1}{g} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \right] \right] \right]$$

$$(b) \ g = \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon}$$

$$(c) \ g > \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \Rightarrow \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon}}{\mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left[1 - \frac{1}{g} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \right] \right] \right]$$

$$\text{Case 3: } g \geq \frac{\varepsilon}{1 + \varepsilon} \Rightarrow \frac{t_{I(A)}}{1 - t_{I(A)}} = \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \left[1 - \frac{w_L^{1+\varepsilon}}{\mu_{1+\varepsilon}(w)} \left[1 + \varepsilon \left[1 - \frac{1}{g} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)} \right] \right] \right]$$

There is again a monotonic relationship between the optimal tax rates and g ; both opportunity egalitarian tax rates decrease as g increases. The interpretation is similar to the one in the previous section. A larger g means a lower (social) disutility of effort, and therefore the social planner is less inclined to discourage labor supply.

The Van de gaer objective function $\varphi_{S(A)}$ (focusing on the smallest opportunity set) is characterized by two regimes. As long as $g < \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)}$, the smallest opportunity set is the one of the highest skilled individuals (cases 1 and 2a). In cases 2c and 3, where the reverse inequality holds, the lowest skilled individuals have the smallest opportunity set. Case 2b reflects the situation where $g = \frac{\varepsilon}{1 + \varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_\varepsilon(e)}$. For that specific value of g , $\varphi_{S(A)} = B$ and therefore $t_{S(A)} = t_{BI}$. At that point, there is a kink in the schedule of the optimal tax as a function of g . After the kink, its slope gets smaller in absolute value.

The Roemer objective function $\varphi_{I(A)}$ (focusing on the intersection of the opportunity sets) has three regimes, and the transition between them is smooth. The resulting optimal tax rate $t_{I(A)}$ coincides with $t_{S(A)}$ in cases 1 and 3. It differs, however, in case 2 in which the advantage functions cross. To understand case 2, note that larger taxes induce everybody to work less. This has a positive effect on the advantage of those with $e > g$ and a negative effect on the advantage of those with $e < g$ (see the discussion following

Eq. (13)). Let us now look first at the special situation where $\varepsilon \rightarrow \infty$, and therefore $g = g(1 + \varepsilon)/\varepsilon$. We are then in case 2a and the opportunity frontiers cross at the point where $e = g$ (see Fig. 2). For $e > g$, both objective functions consider the advantage of the highest skilled; for $e < g$, however, $\varphi_{S(A)}$ considers the advantage of the highest skilled while $\varphi_{I(A)}$ considers the advantage of the lowest skilled. In that range of e -values, an increase in the tax has a negative effect on advantage. This undesirable effect will be larger for the Van de gaer objective function than for the Roemer objective function because $\frac{\partial^2 L}{\partial t \partial w} < 0$ (see Eq. (4)). Therefore, we can expect $t_{I(A)} > t_{S(A)}$. Let us now consider what happens when ε decreases somewhat. In that case and as long as $g < \frac{\varepsilon}{1+\varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_e(e)}$, the Roemer and Van de gaer rules consider different opportunity sets for values of $e < g(1 + \varepsilon)/\varepsilon$. As before, the Van de gaer rule looks at the highest skilled, while the Roemer rule looks at the lowest skilled. Therefore, for $e < g$, the previous reasoning still holds. For $g < e \leq g(1 + \varepsilon)/\varepsilon$, the tax increase has a positive effect on advantage and this effect will be larger for the highest skilled than for the lowest skilled. The former effect dominates the latter, however, so that we still have that $t_{I(A)} > t_{S(A)}$. Things change as soon as $g > \frac{\varepsilon}{1+\varepsilon} \frac{\mu_{1+\varepsilon}(e)}{\mu_e(e)}$. Now the objective function $\varphi_{S(A)}$ concentrates on the opportunity set of the lowest skilled. It therefore differs from the objective function $\varphi_{I(A)}$ for values of $e > g(1 + \varepsilon)/\varepsilon$. This is a region with $e > g$ and where therefore a larger tax has a positive effect on advantage. Because $\frac{\partial^2 L}{\partial t \partial w} < 0$, this (positive) effect will be larger for the highest skilled, i.e., in this case for the Roemer rule. We can therefore again expect $t_{I(A)} > t_{S(A)}$. We can summarize this discussion in the following corollary:

Corollary 4. *If $\frac{\varepsilon}{1+\varepsilon} e_L < g < \frac{\varepsilon}{1+\varepsilon} \Rightarrow t_{I(A)} \geq t_{S(A)}$. For other values of g , $t_{I(A)} = t_{S(A)}$.*

Further insights are gained by comparing the results in this section with those in the previous sections. The next corollary compares objective outcome egalitarian and opportunity egalitarian optimal taxes. We find an analogous pattern as the one described in Corollary 1; introducing the notion of opportunity egalitarianism leads to a decrease in the optimal tax rate.

Corollary 5. *For a given value of g , $t_{E(A)} > t_{I(A)} \geq t_{S(A)}$.*

We can also compare objective and subjective opportunity egalitarian taxes (i.e., the results in Propositions 2 and 6). It was already emphasized that the identification of the worst-off individual may be different in the two cases. Let us therefore focus on case 3 of Proposition 6; in that case, the individual with the lowest skills has the smallest opportunity set both in terms of welfare and in terms of advantage. Subjective and objective opportunity taxes coincide when $g\mu_e(e) = \mu_{1+\varepsilon}(e)$. In that case, the opportunity sets in terms of advantage and subjective utilities are equal. In general, however, this condition will not hold and the term $\varepsilon \left[1 - \frac{1}{g} \frac{\mu_{1+\varepsilon}(e)}{\mu_e(e)} \right]$ can be interpreted as an “advantage” correction factor. We find a similar correction factor in the other cases. Its interpretation is akin to the one described in Section 5.2. If $g > \frac{\mu_{1+\varepsilon}(e)}{\mu_e(e)}$, the correction factor brings the optimal tax rate down thus inducing everybody to work harder.

Fig. 4 also indicates that objective opportunity egalitarianism may conflict with Pareto efficiency. Because the problem is completely analogous to the one sketched in the previous section, we simply refer to the discussion there. More specifically, Pareto

efficiency of the solution can be restored by introducing the additional constraint that the optimal tax rate must be situated in the interval $[t_{\text{MIN}}, t_{\text{E}(V)}]$.

6.3. Some further discussion

Finally, we investigate the way optimal taxes change when the inequality in the distribution of skills or tastes changes. We say that the inequality in the distribution of a variable increases, when the new distribution can be obtained from the old one through a sequence of mean preserving spreads that do not change the support. The next corollary describes the way optimal taxes change when the inequality in the distribution of skills or tastes changes.

Corollary 6. *Inequality and optimal taxes:*

(a) *subjective egalitarian taxes*

If the inequality in skills increases, both subjective outcome and opportunity taxes increase. If the inequality in tastes increases, outcome egalitarian taxes increase if $\varepsilon > 1$, and decrease if $\varepsilon < 1$. Opportunity egalitarian taxes do not depend on the degree of inequality in the taste distribution.

(b) *objective egalitarian taxes*

If $g > \frac{\varepsilon}{1+\varepsilon}$, objective egalitarian taxes react in the same way as subjective egalitarian taxes to changes in inequality. If $g < \frac{\varepsilon}{1+\varepsilon}$, they react in the opposite way.

If $g > \frac{\varepsilon}{1+\varepsilon}$, opportunity egalitarian taxes increase if the inequality in the skill distribution increases. If $g < \frac{\varepsilon}{1+\varepsilon} e_L$, opportunity egalitarian taxes decrease if the inequality in the skill distribution increases. If $\varepsilon < 1$, opportunity egalitarian taxes increase if the inequality in the taste distribution increases.

To interpret these results, note that the optimal tax rates imply that a balance has to be found between the effect of the tax on the level of the basic income and the net effect on labor and earnings of the least well off. The level of inequality in skills or tastes only plays a role through its effect on the extent to which the level of basic income responds to changes in the tax rate. From Eq. (7), we have that

$$\frac{\partial B(t)}{\partial t} = L_0 \mu_\varepsilon(e) \mu_{1+\varepsilon}(w) (1-t)^\varepsilon \left(1 - \frac{t}{1-t} \varepsilon \right)$$

Increases in $\mu_\varepsilon(e)$ or $\mu_{1+\varepsilon}(w)$ will increase the positive effect of changes in t on $B(t)$ as long as $\frac{t}{1-t} < \frac{1}{\varepsilon}$. This is the case for subjective taxes and for objective egalitarian taxes with $g > \frac{\varepsilon}{1+\varepsilon}$. In these cases, the positive effect from taxes on the level of basic income is increased, such that the optimal level of taxes increases with $\mu_\varepsilon(e)$ or $\mu_{1+\varepsilon}(w)$. For objective egalitarian taxes with $g < \frac{\varepsilon}{1+\varepsilon} e_L$, the opposite holds true.

7. Empirical illustration

A simple empirical application may illustrate the theoretical framework and the different egalitarian concepts described in earlier sections. To avoid complications with

Table 1
Description of the sample

No. of observations: 184				
	Mean	Standard deviation	Min	Max
Age	37.73	8.07	25.08	54.83
w_i (in euro)	13.79	5.38	5.82	32.05
$(1-t_i)w_i$ (in euro)	8.47	2.66	4.50	18.57
L_i	37.18	3.73	20	45

household size, we use a sample of single males, coming from the 1992 and 1997 waves of the Belgian socioeconomic panel.²⁰ The data set contains information on gross hourly wages w_i , net hourly wages $(1-t_i)w_i$ and contractual working hours per week L_i .²¹ Some descriptive statistics are given in Table 1.

Within the context of our model, it seems logical to interpret the gross wage w_i as a criterion for the innate skill level of individual i , assumed to be beyond her control. Remember from Eq. (4) that individual labor supply is given by:

$$L_i = (e_i(1 - t_i)w_i)^\varepsilon L_0 \quad (21)$$

Given information on L_i and $(1-t_i)w_i$ and setting L_0 equal to an arbitrary constant, individual preference for leisure e_i can then be calculated for different values of ε . First, all w_i 's are divided by the largest gross wage in the sample w_{\max} to generate a normalized w^* -series with support between $[w_L, 1]$. Then, the values for e_i are generated from Eq. (21) for values of ε equal to 0.06, 0.3 and 1. Finally, all e_i 's are divided by the largest e_i -value e_{\max} to obtain a normalized e^* -series with support between $[e_L, 1]$. At first sight, this procedure yields all the information which is necessary to compute the optimal taxes. There is an important problem, however; these calculated data do not fulfill the assumption that e and w are distributed independently (see column 2 in Table 2²²). Without this assumption, however, our theoretical results do no longer hold. We will therefore first formulate a solution to this problem (Section 7.1).

7.1. Implementing the independence assumption

A possible method to alleviate the unwanted correlation is to regress the w^* 's on a constant and the calibrated e^* 's using OLS:

$$w_i^* = \alpha + \beta e_i^* + \eta_i \quad (22)$$

²⁰Because all formulas derived in Section 2 depart from individual optimising behaviour, this data set will presumably fit the theoretical descriptions better than, e.g., a sample on male heads of household.

²¹The individual specific data on t_i take into account all the details of the Belgian tax system and are calculated through microsimulation.

²²Table 2 gives correlation coefficients. Absence of correlation is a necessary, although not sufficient condition, for statistical independence.

Table 2
Correlation between w^* - and e^* -series

ε	$\text{corr}(w^*, e^*)$	$\text{corr}(w_{\text{resid}}^* e_{\text{resid}}^*)$	$\text{corr}(w_{\text{resid}}^* e_{\text{resid}}^*)_{\text{two rounds}}$
0.06	-0.251	-0.121	-0.028
0.3	-0.814	-0.283	-0.038
1	-0.912	-0.657	0.095

The estimated residuals of this OLS regression $\hat{\eta}_i$ are uncorrelated with e_i^* by construction. We can then construct a new w -series, called w_{resid} :

$$w_{\text{resid},i} = \hat{\alpha} + \hat{\beta} \bar{e}^* + \hat{\eta}_i \tag{23}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the estimated coefficients of Eq. (22) and \bar{e}^* stands for the mean of the normalized effort series. Note that the w_{resid} - and the e^* -series will be uncorrelated. Of course, both series cannot be used directly to calculate the optimal tax rates because we have to make sure that the labor supply equations remain valid. Therefore, employing $w_{\text{resid},i}^*$, $w_{\text{resid},i}^*$'s normalized equivalent, and Eq. (21) for all i , we generate a new e -series, called e_{resid} . After normalizing e_{resid} , the correlation between both series reduces substantially as shown in column 3 of Table 2.

We can repeat this method a number of times in order to reduce the correlation even further. That is, regress w_{resid}^* on a constant and e_{resid}^* and use the resulting residuals to construct a new w -series, which will in turn lead to a new e -series through Eq. (21). The results of performing the method twice are summarized in column 4 of Table 2. Because the correlation now has become almost zero, we will use these latter data for the computation of optimal taxes.

Of course, the proposed method could just as well be reversed. Regress the calibrated e^* 's on a constant and on the w^* 's. Use the estimated coefficients, the mean of the normalized gross wage series \bar{w}^* and the residuals to construct a new e -series, e_{resid} . After normalization, a new w -series can be generated, again by using Eq. (21). The choice between both methods has normative implications; the former method defines the compensation variable w in a precise way and assigns all other factors residually to the responsibility variable e , while the latter method does the reverse. However, when we use the data calculated with the latter method, the optimal tax rates are very similar.

7.2. Results

Once both series are identified, w_L^* and e_L^* can be determined accordingly. Approximating the α th moment of a variable x , $\mu_\alpha(x) = \int_{\bar{x}}^{\bar{x}} x^\alpha f(x) dx$, by its natural estimator $\frac{1}{N} \sum_{\bar{x}} x^\alpha$,

Table 3
Values of e_L^* , t_{BI} , $t_{E(V)}$, $t_{S(V)}$ and t_{med} for $\varepsilon = 0.06, 0.3$ and 1

ε	e_L^*	t_{BI}	$t_{E(V)}$	$t_{S(V)} = t_{(V)}$	t_{med}
0.06	1.65e - 06	0.943	0.927	0.899	0.600
0.3	0.102	0.769	0.763	0.758	0.146
1	0.600	0.500	0.460	0.433	0.016

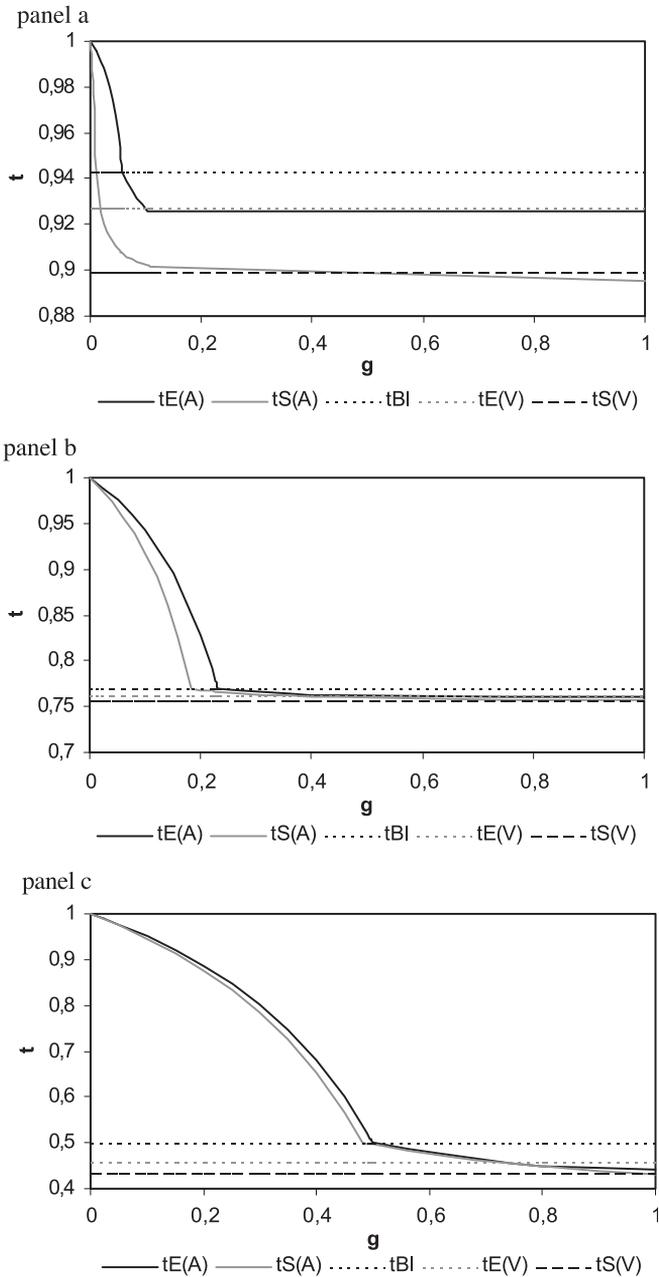


Fig. 5. Empirical results for Belgium. Panel a: $t_{E(A)}$, $t_{S(A)}$, t_{BI} , $t_{E(V)}$ and $t_{S(V)}$ for $\epsilon=0.06$; panel b: $t_{E(A)}$, $t_{S(A)}$, t_{BI} , $t_{E(V)}$ and $t_{S(V)}$ for $\epsilon=0.3$; panel c: $t_{E(A)}$, $t_{S(A)}$, t_{BI} , $t_{E(V)}$ and $t_{S(V)}$ for $\epsilon=1$.

optimal tax rates can be computed for every value of ε . Table 3 presents the calibrated e_L^* , the revenue-maximizing tax rate t_{BI} and the subjective egalitarian optimal taxes $t_{E(V)}$ and $t_{S(V)}$ (here identical to $t_{I(V)}$), computed using the $w_{\text{resid(two rounds)}}^*$ - and $e_{\text{resid(two rounds)}}^*$ -series described above. For comparison purposes, we give in the last column also the tax rates which would be preferred by the median voter in our sample (see Eq. (15)). It is not surprising that the optimal tax rates are high, especially for small values of ε . This is a direct consequence of our use of a social welfare function with infinite inequality aversion.²³ More interesting is the fact that the introduction of opportunity considerations in a subjective framework has a relatively minor influence on the results for our sample.

The objective egalitarian optimal tax rates depend on the chosen value of the social planner's preference for leisure parameter g . Fig. 5 presents $t_{E(A)}$ and $t_{S(A)}$ as a function of g for $\varepsilon = 0.06, 0.3$ and 1 .²⁴ The tax rates t_{BI} , $t_{E(V)}$ and $t_{S(V)}$ are also depicted to allow comparison. It turns out that, after the (first) breakpoint, the objective egalitarian tax rates $t_{E(A)}$ and $t_{S(A)}$ hardly change anymore and are rather close to the subjective egalitarian tax rates. So, there is especially room for discussion about which tax to implement for a government with a large preference for leisure (small value of g). Such a government considers a large part of the labor force as working too hard. Furthermore, remember that Pareto efficiency is violated for tax rates larger than $t_{E(V)}$, which is the case for a range of g -values.

The most striking result is the importance of the efficiency effect, as captured by the labor supply elasticity ε in both subjective and objective egalitarian approaches. It largely determines the level of the optimal taxes (compare the scales in the different panels of Fig. 5) and it has a crucial influence on the level of g at which the breakpoint is situated (this breakpoint level is increasing with ε). For our sample of single males, it can be reasonably assumed that labor supply elasticities are small (perhaps even close to zero). Our results then show that for realistic values of the labor supply elasticities, the egalitarian position will advocate high marginal income tax rates, even after the introduction of opportunity considerations.

8. Conclusion

Much of the traditional optimal taxation literature has concentrated on a model where individuals differ in skills but have identical preferences. This model allows to sidestep some difficult ethical issues and to focus on the tradeoff between "equity" and "efficiency". As soon as one allows for preference differences, one can no longer neglect the deeper question whether it is desirable to compensate individuals for all possible reasons for income (or welfare) differences. While most egalitarian theories would accept that individuals are compensated for differences in (innate) endowments, recent approaches have introduced the

²³Our results are comparable, e.g., to those calculated by [Stern \(1976\)](#) for the maximin case in a model without differences in preferences.

²⁴To simplify the pictures, we do not show the results for $t_{I(A)}$. From Corollary 4, we know that these will lie between $t_{E(A)}$ and $t_{S(A)}$.

importance of responsibility considerations. As a typical example, individuals could legitimately earn a larger income if it results from more effort. Survey research ([Miller, 1992](#)) has shown that this basic intuition is very common among the population at large.

Simple subjective outcome maximin rewards laziness and therefore may go against these basic intuitions. We explore two ways out of this problem. In the first place, we introduce two interpretations of equality of opportunity. [Van de gaer's \(1993\)](#) criterion maximizes the opportunity set of the individuals with the smallest set, the better known criterion of [Roemer \(1998\)](#) focuses on the lower contour set of the opportunity sets. In the subjective version of our model, the two criteria coincide. Not surprisingly, the resulting opportunity optimal taxes are smaller than the outcome egalitarian taxes. In the second place, we propose an “objective” measure of well-being to replace subjective preferences in the social objective function. This so-called advantage function (or “reference preferences”) represents the idea of the social planner about what is a reasonable amount of effort. In this case, the resulting taxes are not necessarily smaller than the subjective egalitarian optimal taxes because they depend on the attitude of the social planner towards effort. Finally, we combine the two intuitions and we analyze the optimal tax rates if the social objective is to equalize opportunities for advantage. This latter model is related to the recent analysis of [Roemer et al. \(2003\)](#), but they work with the extreme case in which the advantage function reduces to income. After the introduction of the advantage function, Van de gaer's criterion of equality of opportunity leads to different optimal taxes than Roemer's criterion.

In our simple model, we can derive closed form solutions for the optimal tax rate under the different interpretations of egalitarianism. These solutions are easily interpretable. Many important problems remain, however. The choice between the different versions of the idea of equality of opportunity remains open. More importantly, our specification of the advantage function is a very primitive one. While good arguments have been given to explore nonwelfarist approaches to optimal taxation and to describe the living standard of individuals in a more objective way than is possible based on subjective preferences, our notion of advantage captures these intuitions only in a rather ad hoc way, focusing as it does on the specification of a “reasonable” amount of effort. Where do these reference preferences come from? For which factors should people be held responsible? Is laziness not a native endowment? And what if maximizing a social welfare function defined in terms of advantage leads to a violation of Pareto efficiency defined in terms of subjective preferences? Is it possible to generalize the specific results from our small empirical illustration to other settings? While all these questions remain open, we firmly believe that the explicit modelling of compensation and responsibility considerations brings us closer to the actual social debate about the pros and cons of progressive income taxation.

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Appendix A

Proof of corollary 3. The continuity of $A(e,w,B,t)$ as a function of e , and Eq. (2) in Lemma 1 imply that advantage is maximal for $g=e$ and that the minima have to be found at the corner points of the domain of e . To find which corner is associated with the minimum level of advantage, note that

$$A(e_L, \bar{w}, B, t) - A(1, \bar{w}, B, t) < 0 \Leftrightarrow e_L^\varepsilon \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e_L \right] - \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \right]$$

$$< 0 \Leftrightarrow e_L^\varepsilon - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e_L^{1+\varepsilon} - 1 + \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} < 0 \Leftrightarrow e_L^\varepsilon - 1 < \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} [e_L^{1+\varepsilon} - 1]$$

Which, because $e_L < 1$ and therefore $e_L^{1+\varepsilon} - 1 < 0$, is equivalent to

$$\frac{e_L^\varepsilon - 1}{e_L^{1+\varepsilon} - 1} > \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \Leftrightarrow g > \frac{\varepsilon}{1 + \varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}.$$

The condition under which $A(e_L, \bar{w}, B, t) - A(1, \bar{w}, B, t) > 0$ can be established by reversing all inequalities.

Proof of proposition 3. Note first that $g \geq \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}$ implies $g \geq \frac{\varepsilon}{1+\varepsilon}$ because $\frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1} > 1$. There is, therefore, a natural ranking in the different cases.

In cases 1–3, $g \frac{1+\varepsilon}{\varepsilon} \geq 1 \geq e$, which by Corollary 2 implies that the lowest advantage level is reached for $w = w_L$. The identification of the level of e leading to the lowest advantage level in cases 1–3, then immediately follows from Corollary 3.

The same corollary also shows that in case 5, the lowest advantage level is reached for $e = 1$. Because at the same time, $g \frac{1+\varepsilon}{\varepsilon} < 1$, we are in case 3 of Corollary 2 and therefore the lowest advantage level is reached for $w = 1$.

The simplified expression in case 4 follows from straightforward calculations.

Proof of proposition 4. Optimal objective egalitarian taxes are found as the maximum of a particular individual’s advantage function. In general terms, the advantage of an individual with characteristics e and w can be written as (using Eqs. (7) and (13)):

$$\tilde{A}(e, w, t) = L_0 t (1 - t)^\varepsilon \mu_\varepsilon(e) \mu_{1+\varepsilon}(w) + L_0 (1 - t)^{1+\varepsilon} w^{1+\varepsilon} e^\varepsilon \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e \right]$$

Maximization of this advantage function w.r.t. t yields as a necessary (first order) condition

$$\frac{t}{1 - t} = \frac{1}{\varepsilon} \left[1 - \frac{(1 + \varepsilon) e^\varepsilon w^{1+\varepsilon}}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e \right] \right]$$

The proposition immediately follows after plugging in the relevant characteristics of the poorest individual, as identified in Proposition 3. In case 2, the individuals with characteristics (e_L, w_L) and $(1, w_L)$ are both worst-off. Maximization of either characteristic's advantage leads to the tax rate given in the proposition. This is easy to verify, keeping in mind that $g = \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}$.

Proof of proposition 5. Cases 1 and 3 follow immediately from Corollary 2 and from the fact that the support of e is given by $[e_L, 1]$.

To derive the result for $\varphi_{I(A)}$ in case 2, start from the definition

$$\varphi_{I(A)} = B + L_0(1 - t)^{1+\varepsilon} \int_{e_L}^1 f_e(e) \min_w \left\{ w^{1+\varepsilon} e^\varepsilon \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e \right] \right\} de$$

The minimum in this expression is found for $w = w_L$ if $1 - \frac{1}{g} \frac{\varepsilon}{1+\varepsilon} e \geq 0$, and therefore $e \leq g \frac{1+\varepsilon}{\varepsilon}$. The minimum is found for $w = 1$ if $e \geq g \frac{1+\varepsilon}{\varepsilon}$. Hence the expression for φ_I .

To establish the result for $\varphi_{S(A)}$ in case 2, start from the definition

$$\varphi_{S(A)} = \min_w \left\{ B + L_0(1 - t)^{1+\varepsilon} w^{1+\varepsilon} \left[\mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{1+\varepsilon}(e) \right] \right\}$$

In case 2a, the term in square brackets becomes negative. The minimum is found when the term in front of these brackets is as large as possible, which is the case if $w = 1$. In case 2c, the term in square brackets is positive, and the expression will be smallest if w is smallest: $w = w_L$. Case 2b is trivial.

Proof of proposition 6. All the results for $t_{S(A)}$ (and for $t_{I(A)}$ in cases 1 and 3) follow immediately from the optimization of Eqs. (19) or (20) after substitution of Eq. (7) for B . The result for $t_{I(A)}$ in case 2 is also obtained after rearranging the first-order conditions resulting from the maximization of the objective function as described in Proposition 5. Note that $C(\cdot)$ is independent of t .

Proof of corollary 4. There is only a difference between $t_{I(A)}$ and $t_{S(A)}$ in case 2 of Proposition 6.

To analyze case 2a, we first rewrite the function $C(\cdot)$ as defined in Proposition 5 as follows

$$C(\cdot) = (w_L^{1+\varepsilon} - 1) \int_{e_L}^{g \frac{1+\varepsilon}{\varepsilon}} e^\varepsilon \left(1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right) f_e(e) de + \mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{\varepsilon+1}(e)$$

Using this result and the expressions in Proposition 6, it follows that

$$\frac{t_{I(A)}}{1 - t_{I(A)}} - \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \frac{1 + \varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} [1 - w_L^{1+\varepsilon}] \int_{e_L}^{g \frac{1+\varepsilon}{\varepsilon}} e^\varepsilon \left[1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right] f_e(e) de$$

It is true in general that $1 > w_L^{1+\varepsilon}$. At the same time, over the range of the integral $e \leq g \frac{1+\varepsilon}{\varepsilon}$ such that the integral will be positive. As a result, in case 2a, $t_{I(A)} \geq t_{S(A)}$.

To analyze case 2c, we rewrite $C(\cdot)$ as

$$C(\cdot) = (1 - w_L^{1+\varepsilon}) \int_{g^{\frac{1+\varepsilon}{\varepsilon}}}^1 e^\varepsilon \left(1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right) f_e(e) de + w_L^{1+\varepsilon} \mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{\varepsilon+1}(e) w_L^{1+\varepsilon}$$

It then follows for case 2c in Proposition 6 that

$$\frac{t_{I(A)}}{1 - t_{I(A)}} - \frac{t_{S(A)}}{1 - t_{S(A)}} = \frac{1}{\varepsilon} \frac{1 + \varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} [w_L^{1+\varepsilon} - 1] \int_{g^{\frac{1+\varepsilon}{\varepsilon}}}^1 e^\varepsilon \left[1 - \frac{e}{g} \frac{\varepsilon}{1 + \varepsilon} \right] f_e(e) de$$

It is obvious that $t_{I(A)} \geq t_{S(A)}$, because again $w_L^{1+\varepsilon} < 1$ and, over the range of e , $1 - \frac{e}{g} \frac{\varepsilon}{1+\varepsilon} \leq 0$, such that the integral is negative.

Case 2b follows from the results for cases 2a and 2c.

Proof of corollary 5. The relationship between $t_{I(A)}$ and $t_{S(A)}$ has already been shown in Corollary 4, so that we only concentrate on the first inequality in the statement of the corollary. We distinguish four cases.

(1) $g \leq \frac{\varepsilon}{1+\varepsilon} e_L$. It follows from Propositions 4 and 6 that

$$\frac{t_{E(A)}}{1 - t_{E(A)}} - \frac{t_{I(A)}}{1 - t_{I(A)}} = -\frac{1}{\varepsilon} \frac{1 + \varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} - \mu_\varepsilon(e) + \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{1+\varepsilon}(e) \right]$$

This expression will be positive if and only if $1 - \mu_\varepsilon(e) < \frac{1}{g} \frac{\varepsilon}{1+\varepsilon} [1 - \mu_{1+\varepsilon}(e)]$. It follows from $g \leq \frac{\varepsilon}{1+\varepsilon} e_L$ that $1 \leq \frac{1}{g} \frac{\varepsilon}{1+\varepsilon}$. Therefore, it is sufficient that $1 - \mu_\varepsilon(e) < 1 - \mu_{1+\varepsilon}(e)$ or that $\mu_\varepsilon(e) > \mu_{1+\varepsilon}(e)$. This is indeed the case.

(2) $\frac{\varepsilon}{1+\varepsilon} e_L \leq g < \frac{\varepsilon}{1+\varepsilon}$. It follows from Propositions 4 and 6 that

$$\frac{t_{E(A)}}{1 - t_{E(A)}} - \frac{t_{I(A)}}{1 - t_{I(A)}} = \frac{1}{\varepsilon} \frac{1 + \varepsilon}{\mu_\varepsilon(e) \mu_{1+\varepsilon}(w)} \left[\frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} - 1 + C(\cdot) \right]$$

From the definition of $C(\cdot)$, we have that

$$C(\cdot) > \int_{g^{\frac{1+\varepsilon}{\varepsilon}}}^1 h(e) f_e(e) de \tag{24}$$

where

$$h(e) = e^\varepsilon - e^{1+\varepsilon} \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \tag{25}$$

It is easily verified that

$$\frac{\partial h(e)}{\partial e} \geq (\leq) 0 \Leftrightarrow e \leq (\geq) g. \tag{26}$$

In the integral in Eq. (24), e goes from $g \frac{1+\varepsilon}{\varepsilon} \geq g$ to 1. Over that range, $\frac{\partial h(e)}{\partial e} \leq 0$, such that $h(1)$ is a minimum over the range of e . Because the weights attached to the values of $h(e)$ integrate to less than 1 and $h(e) \leq 0$, we have that

$$\int_{g \frac{1+\varepsilon}{\varepsilon}}^1 h(e) f_e(e) de > h(1) = 1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \tag{27}$$

Combination of Eqs. (24) and (27) shows that the difference in tax rates is positive.

$$(3) \frac{\varepsilon}{1+\varepsilon} \leq g \leq \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon}-1}{e_L^\varepsilon-1}.$$

Analogously, we derive

$$\frac{t_{E(A)}}{1 - t_{E(A)}} - \frac{t_{I(A)}}{1 - t_{I(A)}} = -\frac{1}{\varepsilon} \frac{(1 + \varepsilon)w_L^{1+\varepsilon}}{\mu_\varepsilon(e)\mu_{1+\varepsilon}(w)} \left[1 - \mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} (1 - \mu_{1+\varepsilon}(e)) \right]$$

which will be non negative if $\left[1 - \mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1+\varepsilon} (1 - \mu_{1+\varepsilon}(e)) \right] \geq 0$, which is equivalent to

$$\mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} \mu_{1+\varepsilon}(e) \geq 1 - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon}$$

which can be written as

$$\int_{e_L}^1 h(e) f_e(e) de \geq h(1) \tag{A}$$

Due to Eq. (26), $h(e)$ will be minimal at either $e = e_L$ or $e = 1$. Consequently,

$$\int_{e_L}^1 h(e) f_e(e) de \geq \text{Min}\{h(e_L), h(1)\} \tag{B}$$

If $\text{Min}\{h(e_L), h(1)\} = h(1) = 1 - \frac{1}{g} \frac{\varepsilon}{1+\varepsilon}$, then the condition for the difference between tax rates to be nonnegative holds trivially. If $\text{Min}\{h(e_L), h(1)\} = h(e_L)$ then in view of Eq. (B), a sufficient condition for Eq. (A) to hold true is that $h(e_L) \geq h(1)$, which reduces to $g \leq \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon}-1}{e_L^\varepsilon-1}$. This is always met for the range of g considered in case (3).

$$(4) \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon}-1}{e_L^\varepsilon-1} < g.$$

In this case

$$\frac{t_{E(A)}}{1 - t_{E(A)}} - \frac{t_{I(A)}}{1 - t_{I(A)}} = -\frac{1}{\varepsilon} \frac{(1 + \varepsilon)w_L^{1+\varepsilon} e_L^\varepsilon}{\mu_\varepsilon(e)\mu_{1+\varepsilon}(w)} \left[1 - e_L^{-\varepsilon} \mu_\varepsilon(e) - \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} (e_L - e_L^{-\varepsilon} \mu_{1+\varepsilon}(e)) \right].$$

This difference in taxes will be positive if and only if

$$1 - e_L^{-\varepsilon} \mu_\varepsilon(e) < \frac{1}{g} \frac{\varepsilon}{1 + \varepsilon} e_L \left[1 - e_L^{-(1+\varepsilon)} \mu_{1+\varepsilon}(e) \right]$$

This inequality can be written as

$$\int_{e_L}^1 h(e) f_e(e) de > e_L^\varepsilon - \frac{1}{g} \frac{\varepsilon}{1+\varepsilon} e_L^{1+\varepsilon} \quad (\text{C})$$

Due to Eq. (26), $h(e)$ will be minimal at either $e = e_L$ or $e = 1$. Consequently, Eq. (B) must hold here as well. If $\text{Min}\{h(e_L), h(1)\} = h(e_L) = e_L^\varepsilon - \frac{1}{g} \frac{\varepsilon}{1+\varepsilon} e_L^{1+\varepsilon}$, then the condition for the difference between tax rates to be nonnegative holds trivially. If $\text{Min}\{h(e_L), h(1)\} = h(1)$ then in view of Eq. (B), a sufficient condition for Eq. (C) to hold true is that $h(e_L) \leq h(1)$, which reduces to $g \geq \frac{\varepsilon}{1+\varepsilon} \frac{e_L^{1+\varepsilon} - 1}{e_L^\varepsilon - 1}$. This is always met for the range of g considered in case 4.

Proof of corollary 6. $\mu_{1+\varepsilon}(w)$ is the expected value of a convex function of w because $\varepsilon > 0$. Similarly, $\mu_\varepsilon(e)$ is the expected value of a convex function of e when $\varepsilon > 1$, but is the expected value of a concave function of e if $\varepsilon < 1$. The results follow immediately from the fact that mean preserving spreads increase the expected value of a convex function, and decrease the expected value of a concave function (see, e.g., [Rothschild and Stiglitz, 1970](#)).

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